

Shear viscosity of CFT plasma at finite coupling

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Abstract

We present evidence for the universality of the shear viscosity of conformal gauge theory plasmas beyond infinite coupling. We comment on subtleties of computing the shear viscosity in effective models of gauge/gravity correspondence rather than in string theory.

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1 Introduction

Various universal features of transport properties of strong coupled gauge theory plasmas were discovered within the framework of gauge theory/string theory correspondence of Maldacena [1,2]. For example, all gauge theory plasmas (various gauge groups, matter content, with or without chemical potentials for conserved $U(1)$ charges, with non-commutative spatial directions) that allow for a dual supergravity description have a universal value of the shear viscosity at infinite 't Hooft coupling [3–8]. Similarly, while not universal, the bulk viscosity of non-conformal gauge theory plasmas (again at infinitely strong coupling and provided the dual holographic description is available) appear to satisfy a universal bound [9].

Much less is known about viscosity of gauge theory plasma at finite coupling: the shear viscosity of $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma at finite coupling was computed in [10–13]¹. The difficulties of extending the analysis of [10–13] to other examples of gauge/string correspondence are both technical and conceptual. On a technical side, one deals with a daunting task of studying quasinormal modes of near-extremal black branes in full ten dimensional type IIB supergravity, including (at least to leading order) higher derivative $\mathcal{O}(\alpha'^3)$ -corrections coming from integrating out massive modes of type IIB string. Conceptually, the problem is that the full set of such $\mathcal{O}(\alpha'^3)$ -corrections is not known presently. Typically, in studies of black brane thermodynamics [16–18] and hydrodynamics [10–13] at $\mathcal{O}(\alpha'^3)$ one takes into account only curvature corrections to type IIB supergravity [19–22]. What is known is that additional terms must be present [23–25] and that they might affect the hydrodynamics of dual gauge theory plasma. Thus, the available results [10,12] for the shear viscosity of $\mathcal{N} = 4$ SYM plasma can receive further corrections from Ramond-Ramond fluxes in the dual string theory description. We emphasize though that such corrections will be additive (to leading order in α'), and contribution of $\mathcal{O}(\alpha'^3)$ terms due to fluxes can be computed on top of type IIB supergravity, excluding purely higher-derivative curvature corrections. Here, we consider only higher-derivative curvature corrections to type IIB supergravity — as a result our conclusions are subject to caveat alluded to above.

In this letter we present evidence for the universal features of conformal gauge theory plasmas at finite 't Hooft coupling. A claim of this type can make sense only

¹It was also computed in some phenomenological models of gauge/gravity correspondence in [14,15].

if a prescription is given as to how to compare different gauge theories². For example, for $SU(N_c)$ $\mathcal{N} = 4$ SYM plasma at 't Hooft coupling $\lambda_{\mathcal{N}=4}$

$$\frac{\alpha'}{L^2} = \frac{1}{\sqrt{\lambda_{\mathcal{N}=4}}}, \quad (1.1)$$

where L is the radius of AdS_5 . Now, for a general CFT (with a dual string theory description) we have [26]

$$\frac{\alpha'}{L^2} = \frac{1}{\sqrt{\lambda_{CFT}}} \sqrt{\frac{a_{\mathcal{N}=4}}{a_{CFT}}}, \quad (1.2)$$

where $a_{\mathcal{N}=4}$ and a_{CFT} are the central charges of the $\mathcal{N} = 4$ SYM and a given CFT correspondingly. We identify two CFT gauge theory plasmas if according to (1.2) they have the same holographic string tension, *i.e.*,

$$\lambda_{CFT_1} a_{CFT_1} = \lambda_{CFT_2} a_{CFT_2}. \quad (1.3)$$

Our evidence for the universality of the shear viscosity of CFT plasma at finite coupling comes from analysis of a shear viscosity of $\mathcal{N} = 1$ superconformal gauge theory plasma of Klebanov-Witten (KW) [27]. Specifically, we show that the full spectrum of shear quasinormal modes of KW plasma at order $\mathcal{O}(\alpha'^3) \sim \mathcal{O}(\lambda_{KW}^{-3/2})$ is identical to that of the $\mathcal{N} = 4$ plasma at order $\mathcal{O}(\lambda_{\mathcal{N}=4}^{-3/2})$. For a specific case of the lowest quasinormal modes, this immediately implies that the ratio of the shear viscosity to the entropy density is the same in KW and $\mathcal{N} = 4$ SYM plasmas. Note that following our convention (1.3), we set

$$\frac{\lambda_{KW}}{\lambda_{\mathcal{N}=4}} = \frac{16}{27}. \quad (1.4)$$

Gauge theory plasma transport analysis in [10–12] and in this letter are done in the the full ten dimensional type IIB supergravity including higher-derivative curvature corrections [19–22]. As a separate computation, we show that similar analysis done in lower dimensional higher-derivative effective action (obtained without proper Kaluza-Klein reduction) could be misleading. While a specific example we present gives the same value of the shear viscosity as in its higher dimensional counterpart, the agreement is accidental and the two models have different spectra of quasinormal modes as well as different higher order hydrodynamic corrections for the dispersion relation of the lowest quasinormal mode. In view of this, it would be interesting to re-examine the claims in [14, 15].

²This was not the issue in either shear viscosity universality or bulk viscosity bound at infinite coupling.

2 KW plasma at finite temperature and 't Hooft coupling

A string theory dual to KW gauge theory is a near-horizon limit of D3 brane on the conifold [27]. We begin with computing leading order α' corrections to near-extremal D3 branes at the conifold, dual to finite temperature KW plasma at order $\mathcal{O}(\lambda^{-3/2})$ in the 't Hooft coupling.

Type IIB supergravity effective action with only higher-derivative curvature corrections takes form

$$I = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4 \cdot 5!}(F_5)^2 + \dots + \gamma e^{-\frac{3}{2}\phi} W + \dots \right], \quad (2.1)$$

$$\gamma = \frac{1}{8}\zeta(3)(\alpha')^3,$$

where

$$W = C^{hmnk} C_{pmnq} C_h^{rsp} C_{rsk}^q + \frac{1}{2} C^{hkmn} C_{pqmn} C_h^{rsp} C_{rsk}^q. \quad (2.2)$$

In (2.1) ellipses stand for $\mathcal{O}(\alpha'^3)$ contribution of Ramond-Ramond fluxes that we neglect here.

We represent ten dimensional background geometry describing γ -corrected black 3-branes on the conifold by the following ansatz

$$\begin{aligned} ds_{10}^2 &= g_{\mu\nu}^{(0)} dx^\mu dx^\nu + c_4^2 e_\psi^2 + c_5^2 \sum_{a=1}^2 (e_{\theta_a}^2 + e_{\phi_a}^2) \\ &\equiv -c_1^2 dt^2 + c_2^2 (dx^2 + dy^2 + dz^2) + c_3^2 dr^2 + c_4^2 e_\psi^2 + c_5^2 \sum_{a=1}^2 (e_{\theta_a}^2 + e_{\phi_a}^2), \end{aligned} \quad (2.3)$$

where $c_i = c_i(r)$. The metric ansatz (2.3) is the most general one, given the $U(1) \times SU(2) \times SU(2)$ symmetries of the KW plasma [27]. The frames $\{e_{\theta_a}, e_{\phi_a}\}$ are defined so that the metric on a unit $T^{1,1}$ (the angular part of the conifold) is³

$$(dT^{1,1})^2 = e_\psi^2 + \sum_{a=1}^2 (e_{\theta_a}^2 + e_{\phi_a}^2). \quad (2.4)$$

For the dilaton we assume $\phi = \phi(r)$ and for the five-form

$$F_5 = \mathcal{F}_5 + \star \mathcal{F}_5, \quad \mathcal{F}_5 = -4 d\text{vol}_{T^{1,1}}. \quad (2.5)$$

³See [28] for explicit expressions.

In (2.5) the 5-form flux is chosen in such a way that $\gamma = 0$ solution corresponds to $c_4 = c_5 = 1$. To leading order in γ we further parameterize

$$\begin{aligned}
c_1 &= r \left(1 - \frac{r_0^4}{r^4} \right)^{1/2} e^{-\frac{5}{3}\nu_1} (1 + a + 4b) , \\
c_2 &= r e^{-\frac{5}{3}\nu_1} , \\
c_3 &= \frac{1}{r \left(1 - \frac{r_0^4}{r^4} \right)^{1/2}} e^{-\frac{5}{3}\nu_1} (1 + b) , \\
c_4 &= e^{\nu_1 - \nu_2} , \\
c_5 &= e^{\nu_1 + \nu_2} ,
\end{aligned} \tag{2.6}$$

where the asymptotic normalizable components of gravitational modes ν_1 and ν_2 are dual to KW plasma operators of dimension-eight and dimension-six correspondingly [28].

Equation of motion for ν_2 determined to order $\mathcal{O}(\gamma^2)$ from (2.1) takes form

$$0 = \nu_2'' + \frac{5r^4 - r_0^4}{r(r^4 - r_0^4)} \nu_2' - \frac{12r^2}{r^4 - r_0^4} \nu_2 . \tag{2.7}$$

The absence of non-normalizable mode asymptotically as $r \rightarrow \infty$ and the regularity at the horizon $r \rightarrow r_{0+}$ determines unique solution to (2.7):

$$\nu_2(r) = 0 . \tag{2.8}$$

Vanishing of ν_2 implies that the remaining modes $\{a, b, \nu_1\}$ are exactly the same⁴ as for the near-extremal D3 branes [16]

$$\begin{aligned}
a &= -\gamma \frac{15r_0^4}{2r^4} \left(25 \frac{r_0^4}{r^4} - 79 \frac{r_0^8}{r^8} + 25 \right) , \\
b &= \gamma \frac{15r_0^4}{2r^4} \left(5 \frac{r_0^4}{r^4} - 19 \frac{r_0^8}{r^8} + 5 \right) , \\
\nu_1 &= \gamma \frac{15r_0^8}{32r^8} \left(1 + \frac{r_0^4}{r^4} \right) .
\end{aligned} \tag{2.9}$$

The dilaton ϕ also receives γ corrections, $\phi \propto \gamma$ [16]. It is easy to see that to order $\mathcal{O}(\gamma)$ gravitational perturbations do not mix with the dilaton perturbation; moreover to study gravitational perturbations we can consistency set $\phi = 0$. The Hawking temperature corresponding to the metric (2.3) is [16]

$$T = T_0 (1 + 15\gamma) \equiv \frac{r_0}{\pi} (1 + 15\gamma) . \tag{2.10}$$

⁴We explicitly verified this.

3 Shear viscosity of KW plasma at finite 't Hooft coupling

A relation between hydrodynamics of gauge theory plasmas and the spectrum of quasinormal modes in dual near-extremal string theory backgrounds is explained in [29]. Computation we need to perform here literally repeats the analysis done in [11] — the only difference being that here we have a slightly more complicated background geometry (2.3). Thus, we move directly to the results of the analysis.

A shear quasinormal mode

$$Z_{shear} = Z_{shear,0} + \gamma Z_{shear,1} + \mathcal{O}(\gamma^2) \quad (3.1)$$

of frequency and momentum $(\omega, q = |\vec{q}|)$ in the background geometry (2.3) satisfies the following equations

$$\begin{aligned} 0 &= Z''_{shear,0} + \frac{x^2 \mathfrak{q}^2 + \mathfrak{w}^2}{x(\mathfrak{w}^2 - x^2 \mathfrak{q}^2)} Z'_{shear,0} + \frac{\mathfrak{w}^2 - x^2 \mathfrak{q}^2}{x^2(1 - x^2)^{3/2}} Z_{shear,0} , \\ 0 &= Z''_{shear,1} + \frac{x^2 \mathfrak{q}^2 + \mathfrak{w}^2}{x(\mathfrak{w}^2 - x^2 \mathfrak{q}^2)} Z'_{shear,1} + \frac{\mathfrak{w}^2 - x^2 \mathfrak{q}^2}{x^2(1 - x^2)^{3/2}} Z_{shear,1} + J_{shear,0}^{KW} , \end{aligned} \quad (3.2)$$

where the source $J_{shear,0}^{KW}$ is a functional of the zero's order shear mode $Z_{shear,0}$

$$\begin{aligned} J_{shear,0}^{KW} &= \mathcal{C}_{shear,KW}^{(4)} \frac{d^4 Z_{shear,0}}{dx^4} + \mathcal{C}_{shear,KW}^{(3)} \frac{d^3 Z_{shear,0}}{dx^3} + \mathcal{C}_{shear,KW}^{(2)} \frac{d^2 Z_{shear,0}}{dx^2} \\ &\quad + \mathcal{C}_{shear,KW}^{(1)} \frac{dZ_{shear,0}}{dx} + \mathcal{C}_{shear,KW}^{(0)} Z_{shear,0} \\ &\equiv \hat{\mathcal{C}}_{shear,KW}^{(1)} \frac{dZ_{shear,0}}{dx} + \hat{\mathcal{C}}_{shear,KW}^{(0)} Z_{shear,0} , \end{aligned} \quad (3.3)$$

where in the second equality we used the first equation in (3.2). As in [11],

$$\mathfrak{w} = \frac{\omega}{2\pi T_0} , \quad \mathfrak{q} = \frac{q}{2\pi T_0} , \quad (3.4)$$

and instead of the radial coordinate r we used

$$x \equiv \left(1 - \frac{r_0^4}{r^4}\right)^{1/2} . \quad (3.5)$$

The connection coefficients $\mathcal{C}_{shear,KW}^{(i)}$ are given explicitly in Appendix A.

Introducing⁵

$$Z_{shear}(x) = x^{-i\mathfrak{w}(1-15\gamma)} z_{shear}(x) , \quad (3.6)$$

⁵The incoming wave boundary conditions must be imposed as explained in [13].

a spectrum of the quasinormal modes is determined solving (3.2) subject to the boundary conditions [29]

$$z_{shear} \Big|_{x \rightarrow 0_+} = 1, \quad z_{shear} \Big|_{x \rightarrow 1_-} = 0. \quad (3.7)$$

Superficially, the quasinormal modes of KW plasma are different from those of the $\mathcal{N} = 4$ SYM plasma. Indeed, given that [11]

$$\mathcal{C}_{shear, \mathcal{N}=4}^{(4)} = 45(1 - x^2)^4 \quad (3.8)$$

is different from $\mathcal{C}_{shear, KW}^{(4)}$ (see (A.1)), as well as all the other coefficients, one is tempted to conclude that

$$J_{shear,0}^{KW} \neq J_{shear,0}^{\mathcal{N}=4}.$$

This is incorrect however. The source term in (3.2) must be evaluated on the zero's order shear mode $Z_{shear,0}$. Using the first equation in (3.2) (which is a supergravity approximation and thus is the same for both KW and $\mathcal{N} = 4$ SYM plasmas) it is straightforward to show that even though $\mathcal{C}_{shear, KW}^{(i)} \neq \mathcal{C}_{shear, \mathcal{N}=4}^{(i)}$,

$$J_{shear,0}^{KW} \equiv J_{shear,0}^{\mathcal{N}=4} \quad (3.9)$$

for all $(\mathfrak{w}, \mathfrak{q})$.

Eq.(3.9) is our main result. It establishes that the full spectrum of shear quasinormal modes is the same in KW and $\mathcal{N} = 4$ SYM plasmas⁶. To some extent the equivalence of the source terms for the KW and the $\mathcal{N} = 4$ CFTs could have been expected given that the scalar mode ν_2 deforming the $U(1)$ fiber inside the $T^{1,1}$ of the dual KW geometry is not excited at $\mathcal{O}(\gamma)$ order, see (2.8). Thus, it is reasonable to expect that the Kaluza-Klein reduction of type IIB string theory on S^5 would be the same as for the rigid $T^{1,1}$ (apart from the obvious rescaling of the five-dimensional gravitational coupling)⁷.

In order to relate to the analysis of the next section we reproduce from [11, 13] the lowest quasinormal mode of (3.2) and its dispersion relation:

$$\begin{aligned} z_{shear,0} &= z_{shear,0}^{(0)} + i\mathfrak{q}z_{shear,0}^{(1)} + \mathcal{O}(\mathfrak{q}^2), \\ z_{shear,1} &= z_{shear,1}^{(0)} + i\mathfrak{q}z_{shear,1}^{(1)} + \mathcal{O}(\mathfrak{q}^2), \end{aligned} \quad (3.10)$$

⁶We believe the same statement is correct for the spectrum of sound quasinormal modes as well.

⁷We expect a similar argument to apply to more general examples of $AdS_5 \times SE^5/CFT_4$ correspondence.

$$\begin{aligned}
z_{shear,0}^{(0)} &= 1, & z_{shear,0}^{(1)} &= \frac{1}{2} \frac{\mathfrak{q}}{\mathfrak{w}} x^2, \\
z_{shear,1}^{(0)} &= \frac{25}{16} x^2 (x^4 - 4x^2 + 5), \\
z_{shear,1}^{(1)} &= -\frac{1}{32\mathfrak{q}\mathfrak{w}} x^2 \left(\mathfrak{q}^2 (-240 - 1565x^2 - 860x^4 + 695x^6) \right. \\
&\quad \left. + 16\mathfrak{w}^2 (594 - 264x^2 + 43x^4) \right) - \frac{15\mathfrak{q}}{2\mathfrak{w}} x^2.
\end{aligned} \tag{3.11}$$

Imposing the Dirichlet condition on $z_{shear,0}$ at the boundary determines the lowest shear quasinormal frequency

$$\mathfrak{w} = -i \Gamma_\eta \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3), \quad \Gamma_\eta = \frac{1}{2} + \frac{105}{2} \gamma + \mathcal{O}(\gamma^2). \tag{3.12}$$

4 Shear viscosity in effective higher-derivative gauge/gravity models

Computation of the quasinormal modes in near extremal backgrounds of ten dimensional type IIB supergravity including $\mathcal{O}(\alpha'^3)$ higher-derivative corrections is technically rather involved. One difficulty is a high dimensionality of the background space-time. For the latter reason it is desirable to do the computations in lower dimensional effective description. Of course, had a relevant Kaluza-Klein (KK) reduction of the higher-derivative type IIB supergravity been known, both ways to do the computations are equivalent. In the absence of a consistent KK reduction one usually resorts to guesses as to what the lower dimensional effective action would look like. In this section we present one example which illustrates pitfalls of such an approach.

Suppose we would like to guess an effective action corresponding to (2.1) KK reduced on S^5 (or $T^{1,1}$). As a minimal requirement we would ask that the near extremal background of our effective action faithfully reproduces the thermodynamics of the full ten dimensional background. A natural guess (satisfying the minimal requirement) would then be

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R + 12 + \gamma W \right], \tag{4.1}$$

where W is given again by (2.2). Note that (4.1) is precisely the effective action used in [16] to describe α' corrections of the near extremal D3 branes.

Repeating the analysis of the shear quasinormal modes in this case we find the same

set of equations (3.2), albeit with the different source term:

$$\begin{aligned}
J_{shear,0}^{eff} &= \mathcal{C}_{shear,eff}^{(4)} \frac{d^4 Z_{shear,0}}{dx^4} + \mathcal{C}_{shear,eff}^{(3)} \frac{d^3 Z_{shear,0}}{dx^3} + \mathcal{C}_{shear,eff}^{(2)} \frac{d^2 Z_{shear,0}}{dx^2} \\
&\quad + \mathcal{C}_{shear,eff}^{(1)} \frac{dZ_{shear,0}}{dx} + \mathcal{C}_{shear,eff}^{(0)} Z_{shear,0} \\
&\equiv \hat{\mathcal{C}}_{shear,eff}^{(1)} \frac{dZ_{shear,0}}{dx} + \hat{\mathcal{C}}_{shear,eff}^{(0)} Z_{shear,0} .
\end{aligned} \tag{4.2}$$

The connection coefficients $\mathcal{C}_{shear,eff}^{(i)}$ are given explicitly in Appendix B. Here, $J_{shear,0}^{eff}$ is actually different from $J_{shear,0}^{KW}$. Indeed, using the first equation in (3.2) we find

$$\begin{aligned}
0 \quad \neq \quad J_{shear,0}^{KW} - J_{shear,0}^{eff} &= -\frac{25}{4}x(3x^4 - 8x^2 + 5) \frac{dZ_{shear,0}}{dx} \\
&\quad - \frac{25}{4(\mathfrak{q}^2x^2 - \mathfrak{w}^2)} \left(2\mathfrak{q}^2x^4(3x^2 - 4) - \mathfrak{w}^2(9x^4 - 16x^2 + 5) \right) Z_{shear,0} .
\end{aligned} \tag{4.3}$$

We can explicitly compute the lowest quasinormal mode:

$$\begin{aligned}
z_{shear,0}^{(0)} &= 1 , \quad z_{shear,0}^{(1)} = \frac{1}{2} \frac{\mathfrak{q}}{\mathfrak{w}} x^2 , \\
z_{shear,1}^{(0)} &= 0 , \\
z_{shear,1}^{(1)} &= -\frac{x^2}{2\mathfrak{q}\mathfrak{w}} \left(\mathfrak{q}^2(45x^6 - 60x^4 - 90x^2 - 15) + \mathfrak{w}^2(-264x^2 + 43x^4 + 594) \right) \\
&\quad - \frac{15\mathfrak{q}}{2\mathfrak{w}} x^2 .
\end{aligned} \tag{4.4}$$

While (4.4) is clearly different from (3.11), nonetheless, it has the same dispersion relation (3.12).

Given (4.3), the full shear quasinormal spectrum of $\mathcal{N} = 4$ SYM plasma is different from the one corresponding to the purported holographic dual to (4.1). The difference exists even for the lowest quasinormal mode, although in higher orders in the hydrodynamic approximation.

5 Conclusion

In this letter we conjectured universality of the shear viscosity to entropy ratio of different CFT plasmas at finite 't Hooft coupling. While we presented only one example for a comparison with the $\mathcal{N} = 4$ SYM plasma, namely that of Klebanov-Witten plasma and only to leading order in the inverse 't Hooft coupling, the fact that the full shear quasinormal spectra of both plasmas agree suggests that the agreement extends to

other CFT plasmas as well, and probably also beyond leading inverse 't Hooft coupling correction.

Proving our conjecture to all orders in α' (or the inverse 't Hooft coupling) is unlikely to be possible. We believe that at least to order $\mathcal{O}(\lambda^{-3/2})$ the universality can be established by carefully analyzing Kaluza-Klein reduction of (2.1) on five dimensional Einstein-Sasaki manifolds. A progress on this issue can also be made by studying CFT plasmas in a boost-invariant setting, as in [12]. We hope to report on this elsewhere.

Within the supergravity approximation (equivalently at infinite 't Hooft coupling) the universality of the shear viscosity to the entropy density ratio extends to gauge theory plasmas in various dimensions. We conjectured here that the finite 't Hooft coupling corrections are universal for all four dimensional CFTs, which allow for a dual string theory description. It would be interesting to explore such corrections for conformal gauge theory plasmas in spacetime dimensions other than four.

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Appendix

A Coefficients of $J_{shear,0}^{KW}$

$$\mathcal{C}_{shear,KW}^{(4)} = \frac{(1-x^2)^2}{\mathfrak{q}^2 x^2 - \mathfrak{w}^2} \left(\mathfrak{q}^2 (45x^6 - 90x^4 + 57x^2) - \mathfrak{w}^2 (37x^4 - 74x^2 + 49) \right) \quad (\text{A.1})$$

$$\mathcal{C}_{shear,KW}^{(3)} = \frac{2(x^2 - 1)}{(\mathfrak{q}^2 x^2 - \mathfrak{w}^2)^2 x} \left(\mathfrak{q}^4 (315x^{10} - 585x^8 + 261x^6 + 57x^4) - 8x^2 \mathfrak{w}^2 \mathfrak{q}^2 (85x^6 - 173x^4 + 103x^2 - 3) + \mathfrak{w}^4 (333x^6 - 703x^4 + 467x^2 - 49) \right) \quad (\text{A.2})$$

$$\begin{aligned} \mathcal{C}_{shear,KW}^{(2)} = & -\frac{2}{x^2(\mathfrak{q}^2 x^2 - \mathfrak{w}^2)\sqrt{1-x^2}} \left(\mathfrak{q}^4 (83x^{10} - 249x^8 + 237x^6 - 71x^4) \right. \\ & \left. + 2\mathfrak{q}^2 \mathfrak{w}^2 x^2 (x^2 - 1)(25x^4 - 50x^2 + 37) + \mathfrak{w}^4 (1 - x^2)(37x^4 - 74x^2 + 49) \right) \\ & + \frac{1}{8(\mathfrak{q}^2 x^2 - \mathfrak{w}^2)^3 x^2} \left(\mathfrak{q}^6 x^6 (14015x^8 + 3122x^2 - 22940x^6 + 5203x^4 + 1368) \right. \\ & - 3x^4 \mathfrak{w}^2 \mathfrak{q}^4 (16711x^8 - 30396x^6 + 10243x^4 + 5594x^2 - 1384) \\ & + x^2 \mathfrak{q}^2 \mathfrak{w}^4 (-3864 + 60493x^8 - 127764x^6 + 75993x^4 - 2554x^2) - \mathfrak{w}^6 (22327x^8 \\ & \left. - 51324x^6 + 38179x^4 - 392 - 8022x^2) \right) \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \mathcal{C}_{shear,KW}^{(1)} = & -\frac{2}{x^3(\mathfrak{q}^2 x^2 - \mathfrak{w}^2)^2 \sqrt{1-x^2}} \left(\mathfrak{q}^6 x^6 (332x^6 - 581x^4 + 166x^2 + 71) \right. \\ & - 3x^4 \mathfrak{q}^4 \mathfrak{w}^2 (188x^6 - 509x^4 + 454x^2 - 145) - 3x^2 \mathfrak{q}^2 \mathfrak{w}^4 (100x^6 - 155x^4 + 10x^2 + 57) \\ & \left. + \mathfrak{w}^6 (148x^6 - 259x^4 + 74x^2 + 49) \right) \\ & - \frac{1}{8(\mathfrak{q}^2 x^2 - \mathfrak{w}^2)^4 x^3} \left(\mathfrak{q}^8 x^8 (8333x^4 + 3122x^2 + 1368 - 17580x^6 + 5525x^8) \right. \\ & - 2x^6 \mathfrak{q}^6 \mathfrak{w}^2 (8582x^2 - 5040 - 39148x^6 + 25791x^4 + 9983x^8) + 4x^4 \mathfrak{q}^4 \mathfrak{w}^4 (-2148 \\ & - 30048x^6 + 32523x^4 - 6170x^2 + 4943x^8) + 2x^2 \mathfrak{q}^2 \mathfrak{w}^6 (816 + 17556x^6 - 30025x^4 \\ & \left. + 11286x^2 + 2935x^8) - \mathfrak{w}^8 (-7980x^6 + 2217x^4 + 234x^2 + 392 + 7105x^8) \right) \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned}
C_{shear,KW}^{(0)} = & \frac{1}{8x^4(\mathfrak{q}^2x^2 - \mathfrak{w}^2)(1-x^2)^{3/2}} \left(\mathfrak{q}^4x^6(5621x^2 - 1370 - 7540x^4 + 3385x^6) \right. \\
& - 2x^2\mathfrak{q}^2\mathfrak{w}^2(-7244x^6 + 3917x^4 + 1726x^2 - 1568 + 3265x^8) + \mathfrak{w}^4(-1568 - 6692x^6 \\
& + 4709x^4 + 982x^2 + 2665x^8) \Big) \\
& + \frac{1}{4x^4(1-x^2)(\mathfrak{q}^2x^2 - \mathfrak{w}^2)} \left(-4\mathfrak{q}^6x^6(19x^4 - 38x^2 + 7) + 4x^4\mathfrak{q}^4\mathfrak{w}^2(x^2 + 5)(x^2 - 7) \right. \\
& + 2x^2\mathfrak{q}^2(25x^6(x^2 - 1)(3x^2 - 4) + 2\mathfrak{w}^4(91 + 55x^4 - 110x^2)) - \mathfrak{w}^2(4\mathfrak{w}^4(37x^4 - 74x^2 \\
& + 49) + 25x^4(x^2 - 1)(9x^4 - 16x^2 + 5)) \Big)
\end{aligned} \tag{A.5}$$

B Coefficients of $J_{shear,0}^{eff}$

$$C_{shear,eff}^{(4)} = \frac{32(1-x^2)^4(5\mathfrak{q}^2x^2 - 3\mathfrak{w}^2)}{3(\mathfrak{q}^2x^2 - \mathfrak{w}^2)} \tag{B.1}$$

$$C_{shear,eff}^{(3)} = \frac{64(x^2 - 1)^3}{3x(\mathfrak{q}^2x^2 - \mathfrak{w}^2)^2} \left(5\mathfrak{q}^4x^4(7x^2 + 1) - 2x^2\mathfrak{q}^2\mathfrak{w}^2(35x^2 - 3) + 3\mathfrak{w}^4(9x^2 - 1) \right) \tag{B.2}$$

$$\begin{aligned}
C_{shear,eff}^{(2)} = & -\frac{64(x^2 - 1)^3}{3(\mathfrak{q}^2x^2 - \mathfrak{w}^2)x^2\sqrt{1-x^2}} \left(7\mathfrak{q}^4x^4 + 5x^2\mathfrak{q}^2\mathfrak{w}^2 - 3\mathfrak{w}^4 \right) \\
& + \frac{x^2 - 1}{3x^2(\mathfrak{q}^2x^2 - \mathfrak{w}^2)^3} \left(20\mathfrak{q}^6x^6(x^2 - 1)(307x^4 + 107x^2 + 24) - x^4\mathfrak{q}^4\mathfrak{w}^2(21653x^6 \right. \\
& - 18675x^4 - 4833x^2 + 2080) + 2x^2\mathfrak{q}^2\mathfrak{w}^4(12379x^6 - 15043x^4 + 2585x^2 + 304) \\
& \left. - 3\mathfrak{w}^6(2399x^6 - 3089x^4 + 733x^2 + 32) \right)
\end{aligned} \tag{B.3}$$

$$\begin{aligned}
\mathcal{C}_{shear,eff}^{(1)} = & -\frac{64(1-x^2)^{3/2}}{3(\mathfrak{q}^2x^2-\mathfrak{w}^2)^2x^3} \left(7\mathfrak{q}^6x^6(4x^2+1) - 2x^4\mathfrak{q}^4\mathfrak{w}^2(23x^2-18) \right. \\
& - 10x^2\mathfrak{q}^2\mathfrak{w}^4(3x^2+1) + 3\mathfrak{w}^6(4x^2+1) \Big) \\
& - \frac{1}{3x^3(\mathfrak{q}^2x^2-\mathfrak{w}^2)^4} \left(20\mathfrak{q}^8x^8(x^2-1)(145x^6-254x^4-83x^2-24) - x^6\mathfrak{q}^6\mathfrak{w}^2(7103x^2 \right. \\
& - 4160 - 34756x^6 + 21582x^4 + 9781x^8) + x^4\mathfrak{q}^4\mathfrak{w}^4(-896 - 49012x^6 + 56106x^4 \\
& - 15135x^2 + 7587x^8) + x^2\mathfrak{q}^2\mathfrak{w}^6(448 + 4172x^6 - 13530x^4 + 4875x^2 + 5385x^8) \\
& \left. - 3\mathfrak{w}^8(-560x^6 - 54x^4 + 32 + 67x^2 + 665x^8) \right)
\end{aligned} \tag{B.4}$$

$$\begin{aligned}
\mathcal{C}_{shear,eff}^{(0)} = & -\frac{1}{3(\mathfrak{q}^2x^2-\mathfrak{w}^2)x^4\sqrt{1-x^2}} \left(15\mathfrak{q}^4x^6(59x^4+33-77x^2) - 2x^2\mathfrak{q}^2\mathfrak{w}^2(925x^6 \right. \\
& + 384 - 1019x^4 - 65x^2) + 3\mathfrak{w}^4(101x^2 + 375x^6 + 128 - 529x^4) \Big) \\
& + \frac{32(x^2-1)(\mathfrak{q}^2x^2-\mathfrak{w}^2)(\mathfrak{q}^2x^2+3\mathfrak{w}^2)}{3x^4}
\end{aligned} \tag{B.5}$$

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